Propagation of Current on Bent Wire Antennas—An Experimental Study

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Abstract—**The results of precise measurements of the driving point responses to a 200 ps input pulse of bent wire antennas of circular cross section mounted above a ground plane are presented. Transmission and reflection coefficients (plotted as functions of frequency to 3 GHz) describing the propagation and reflection of monochromatic current waves on the bent wires are inferred from the measurements. The results show that the current pulse reflected from a sharp bend ($r\theta \ll c\tau_1$), where r is the bend radius, θ is the bend angle, c is the speed of light, and τ_1 is the pulse duration) is a compressed replica of the incident pulse, and that a current pulse propagating around a gradual bend ($r\theta \approx c\tau_1$) is continuously reflected from the whole length of the bend. The time domain measurements and the inferred frequency domain data give a clear physical picture of the mechanism of current propagation on bent-wire antennas.**

I. INTRODUCTION

CURRENT on bent wires was first studied by Pocklington [1] who demonstrated that current propagates along the surface of a perfectly conducting wire of circular cross section approximately with the velocity of light. Hallen [2] examines the vector potential at the surface of a straight perfectly conducting antenna of arbitrary constant cross section, which is excited symmetrically with respect to its axis, and shows that it propagates *exactly* at the velocity of light, without attenuation, dispersion, or reflection. This is not true for a bent antenna. Bates [4]-[6] finds that for narrow strips, the vector potential propagates along the strip virtually without reflection and almost at the velocity of light, provided that the radius of curvature of the strip is constant. The major effect of the curvature of the strip is to introduce attenuation: there is only a small amount of dispersion. Reflection occurs from the points where the radius of curvature changes.

The behavior of the current is different from that of the vector potential. A monochromatic current wave propagating along a straight wire antenna is advanced in phase with respect to the potential, but the phase difference diminishes as the distance from the driving point (or any discontinuities) increases [2, ch. 35], [7]. The phase velocity of the current wave is slower than that of light, which it approaches asymptotically as the distance from the driving point increases. This distance depends upon the electrical thickness of the wire [7]. Thin wires are dispersive only near their driving points or near sharp bends. By assuming that the behavior of the current and the vector potential are similar, Bates [4] obtained qualitative confirmation of his theory for the vector potential by observing the reflections of narrow current pulses from thin wires containing circular bends and calculating current pulse-reflection coefficients.

Bates' coefficients are strong functions of the pulse duration, and the waveforms [6] show that the shape of the pulse reflected from the wire tip changes as the wire is bent. Thus reflection and transmission of current on bent wire antennas is frequency sensitive. A pulse-reflection coefficient for frequency sensitive discontinuities can be misleading [8], [9]. The spectrum of the pulse applied to the bend depends upon distortion at the base region. Changing the wire diameter but using the same input pulse and coaxial line results in a modified pulse being applied to the wire. It is unlikely that the reflection and transmission characteristics for the bends have the same dependence on the wire diameter as the base region, hence pulse reflection and transmission coefficients for different wire diameters should not be directly compared.

Section II describes an accurate method of measuring the reflections of a narrow pulse from a bent wire antenna and presents measurements for both sharp and gradual bends. Reflection coefficients and transmission coefficients that describe the propagation of monochromatic current waves on bent wire antennas are presented in Section III. Section IV discusses the responses and presents a physical explanation of the mechanism of the current flow on a bent wire antenna.

In this paper, the actual quantities measured are voltage pulses in the coaxial line feeding the antenna, and these are simply related to the current pulses by the line impedance. On the antenna, however, voltage and current are related by differential equations, and it is physically more meaningful to refer to current pulses on the antenna. Consequently we refer to current on the antenna and voltage in the transmission line. Characteristics of the current propagation on the antenna are inferred from voltage measurements made in the transmission line.

II. THE DRIVING POINT PULSE RESPONSES OF BENT WIRE ANTENNAS

The geometry of a bent wire antenna and a ground plane is shown in Fig. 1. L_1 is approximately equal to $2L_2$ so that secondary reflections from the bend do not overlap the tip reflection. The driving point responses up to and including the first tip reflection of a number of bent wire antennas were measured on a Tektronix 1S2 TDR. The input signal was a 120 mV, 200 ps pulse (measured between −6 dB points) formed from the step output of the TDR by a short-circuited stub [10]. The spatial length of the pulse was much less than the wire length ($c\tau_1$ where τ_1 is the pulse duration $\cong$ 6 cm). The signals were scanned and averaged using the three-point scanning method of sampling [11], [12] until the rms noise level was reduced to 10 μV. The sampling interval T was 25 ps. A 14-bit D/A and a 12-bit A/D converter was used.

The amplitudes of most of the signals reflected from the bends are too small to be measured directly because of the ripples caused by multiple reflections of the input pulse and

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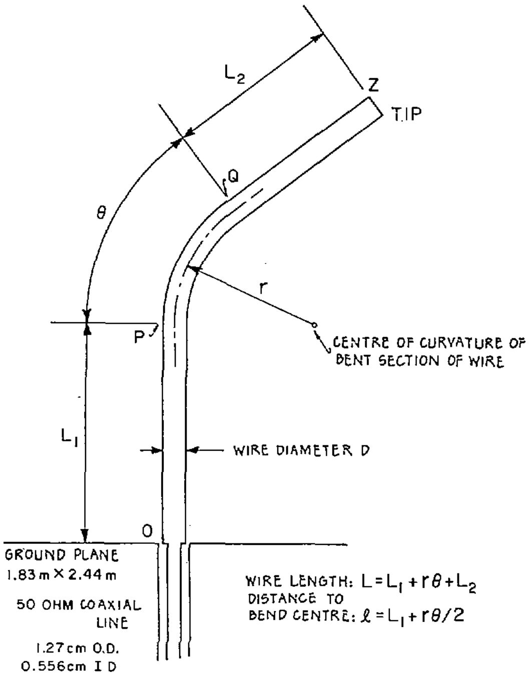
L2
Z
TIP
Q
θ
r
P
CENTRE OF CURVATURE OF BENT SECTION OF WIRE
L1
WIRE DIAMETER D
O
GROUND PLANE 1.83 m × 2.44 m
50 OHM COAXIAL LINE
1.27cm O.D.
0.556cm I D
WIRE LENGTH: L = L1 + rθ + L2
DISTANCE TO BEND CENTRE: ℓ = L1 + rθ/2

Fig. 1. Geometry of bent wire antenna above horizontal ground plane.

the first base reflection from line irregularities. The reflected signal from the bend is obtained indirectly by subtracting the signal reflected from a long straight wire from the signal reflected from the bent wire. This increases the effective noise level of the measurement, but the reflections cannot be isolated satisfactorily unless the ripples are subtracted.

A. Pulse Reflection From Sharp Bends

Fig. 2 shows the reflected signals from straight and sharply bent ($r \cong D$; $r\theta \ll c\tau_1$) wires. The reflections from the bends for $\theta < 80^\circ$ are seen to be smaller than the ripple amplitude. The improvement obtained by subtracting the ripples (i.e., the response shown in Fig. 2(a)) is shown in Fig. 3. The subtraction increases the rms noise level on the "ripple reduced" signals to 20 μV. The residue of the base reflection is everywhere less than 250 μV in magnitude. The slope of the leading edge of the pulse reflected from the base is 342 mV/ns; thus timing jitter is estimated to be about 0.75 ps.

The amplitude and durations (measured at the -6 dB points) of the pulses reflected from the wires summarized in Table I show that the pulse reflected from the end of the straight wire is longer than the pulse reflected from the base. Thus dispersion at the base stretches the pulse, in agreement with Ross [13]. The pulses reflected from the bends are narrower than both the base reflection and the tip reflection. Thus sharp bends compress pulses on reflection. The small positive peaks (marked *S*) following the main negative reflection in Fig. 3(e) and 3(f) are similar to the sidelobes exhibited by pulse compression radar signals [14]. The current pulse-reflection coefficients included in Table I (which are obtained by dividing the peak amplitude of the reflection from the bend by the peak amplitude of the reflection from the tip of a straight wire and changing its sign to account for the difference between current and voltage reflection coefficients), although in good agreement with those reported by Bates [4], are only valid for a 200 ps input pulse [8].

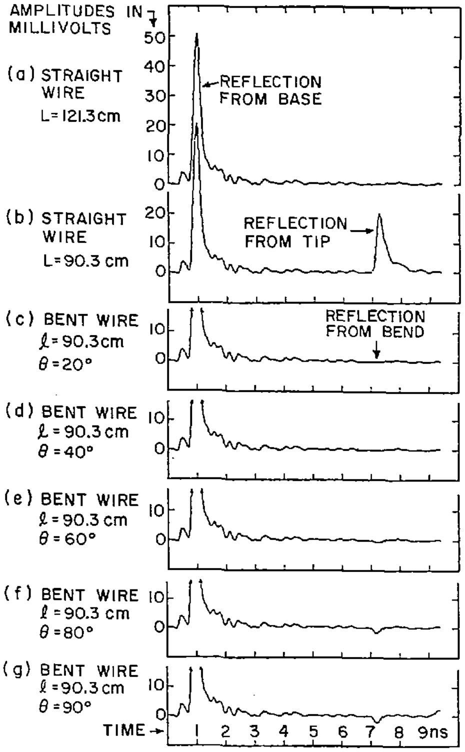

Fig. 2. Measured driving point responses to 200 ps input pulse of straight and sharply bent (r = 3 mm) 0.32 cm diameter wires.

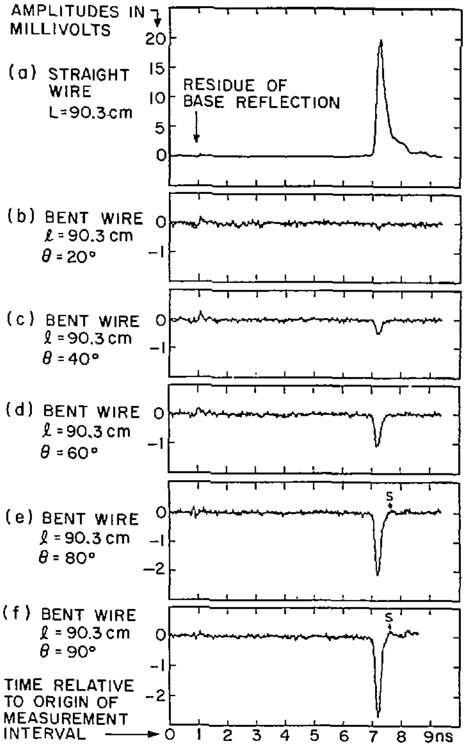

Fig. 3. Driving point responses of straight and sharply bent (r = 3 mm) 0.32 cm diameter wires after subtracting ripples.

TABLE I
PARAMETERS OF PULSES REFLECTED FROM STRAIGHT AND SHARPLY (r = 3MM) BENT 0.32 CM DIAMETER WIRES

	Pulse amplitude (millivolts)	Pulse duration measured between -6 dB points (picoseconds)	Current pulse reflection coefficient
base of wire	50.5	257 ± 12	-
Tip of straight wire	19.85	306 ± 12	-
θ=20° bend	-0.2	208 ± 12	0.01
θ=40° bend	-0.48	208 ± 12	0.024
θ=60° bend	-1.08	208 ± 12	0.054
θ=80° bend	-2.18	208 ± 12	0.110
θ=90° bend	-2.71	208 ± 12	0.136

B. Pulse Reflection from Gradual Bends

Fig. 4 shows the reflected signals from gradually bent ($r \gg D$, $r\theta \cong c\tau_1$) wires after the ripples have been subtracted. The rms noise on the "ripple reduced" signals of Fig. 4(e), (f) was reduced to about 3 μV, and a small amount of ripple remains on the signals. This is caused by secondary reflections of the wire tip reflection from the coaxial line irregularities. Timing jitter remains at 0.75 ps; however, the quantizing error in the voltage scanning the sampling oscilloscope was found to be 0.6 ps, so this error could not be reduced.

The arrival times of signals (assuming free space velocity) corresponding to the points P and Q in Fig. 1 are marked t_p and t_Q in Fig. 4. The pulse is continuously reflected as it travels around the bend. The duration of the reflection exceeds (t_Q-t_p) only by the pulse duration, and for the large bend angles the reflection is a good approximation to the impulse response since $r\theta > c\tau_1$. The shape is approximately an asymmetrical triangle, although Fig. 4(d) and (e) suggest that the leading edge is slightly curved (this curvature was not caused by a noncircular bend: r was everywhere within ±0.5 mm). The effect of tip radiation is observed in Fig. 4(e).

No significant reflections are observed at times corresponding to the rate of change of curvature of the bend (i.e., at t_p or t_Q) as predicted [4].

C. Pulse Transmission along Bent Wires

Figs. 5-7 show the reflections from the tips of straight and bent wires after the ripples have been subtracted. The time t_s which is marked on all the plots marks the beginning time of the reflection from the tip of the straight wire. As the bend angle increases, the leading edge of the pulse advances in time indicating that energy is coupled across the bend. The times corresponding to return propagation directly across OZ are marked t_{OZ}, and t_a, the arrival time of the earliest detectable signal, is about halfway between t_{OZ} and t_s. This suggests that significant energy is coupled across OZ only in one direction: propagation is essentially along the wire in the other direction. The radiation which occurs when a current pulse traveling at (or near) the velocity of light is incident upon a discontinuity in a wire antenna is directed essentially in the direction of propagation of the pulse [3, p. 147]. This effect has been measured by Ross, *et al.* [5]. It follows that significant coupling occurs across OZ only when the pulse propagates out-

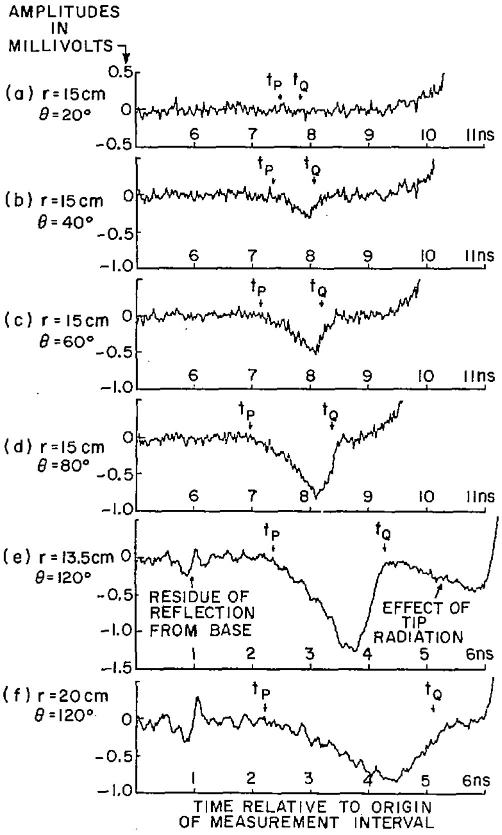

Fig. 4. Reflections from gradual bends in 0.64 cm diameter wires.

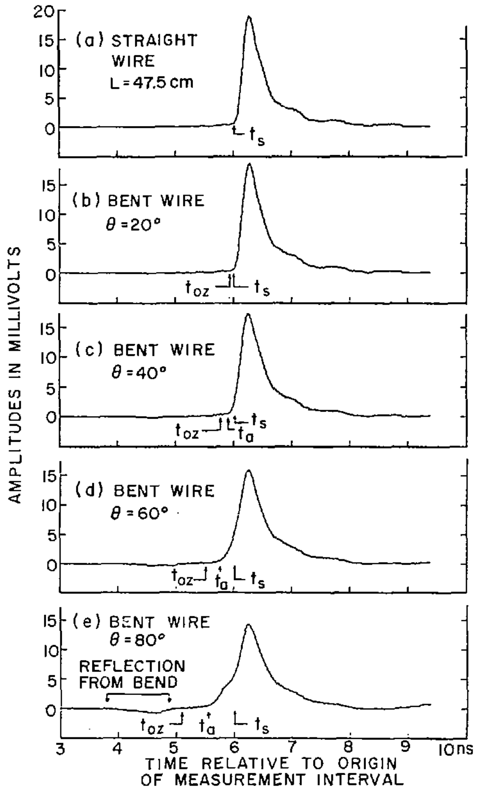

Fig. 5. Reflections from tips of straight and bent (r = 15 cm) 0.64 cm diameter wires 75.7 cm long.

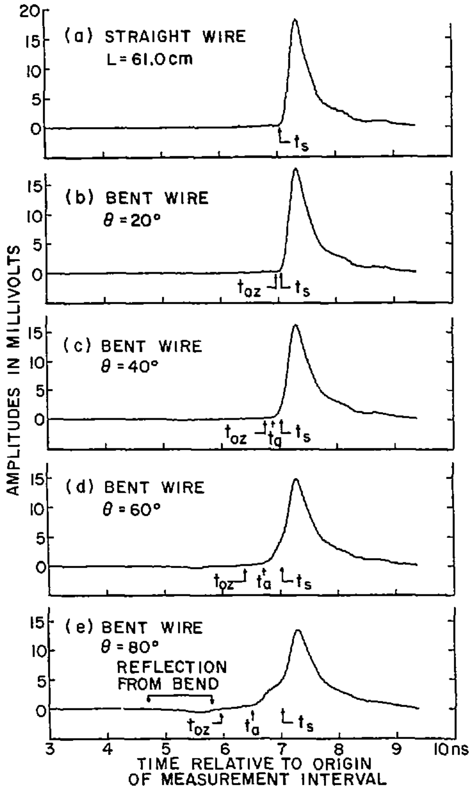

Fig. 6. Reflections from tips of straight and bent (r = 15 cm) 0.64 cm diameter wires 90.9 cm long.

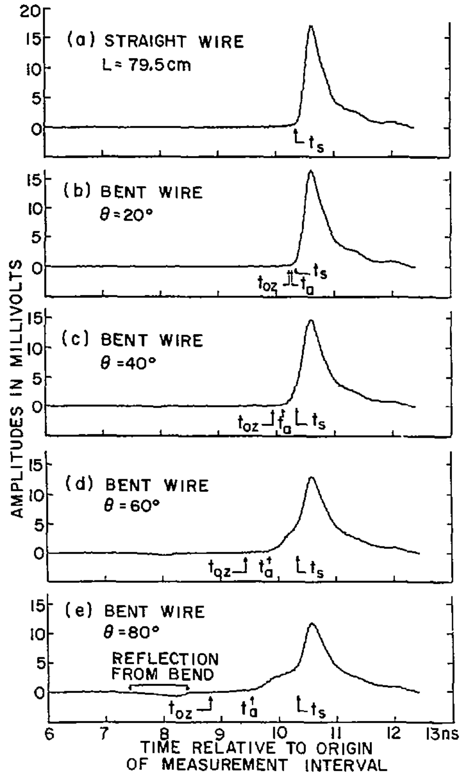

Fig. 7. Reflections from tips of straight and bent (r = 15 cm) 0.64 cm diameter wires 121.3 cm long.

ward from the driving point: return propagation is essentially along the wire. However, if the bend angle becomes large, then a sensible amount of the tip radiation (which is negative for a positive input pulse [8], [16]) is received at the base (this is observed in the measurement of Fig. 4(e)).

The field radiated from the base is essentially a replica of the input pulse [15], [16]. The direction of the current pulse, induced on the bent section of the wire after a positive current pulse (corresponding to the positive input voltage pulse used for the measurements reported in this paper) flows onto the base of the antenna, adds to the current propagating along the wire. The induced current flows both toward the wire tip and back toward the driving point. That which flows toward the driving point is spread out in time because the pulse-shaped field inducing the current is propagating away from the driving point, and is consequently too small to be detected in the measurements reported here. The induced current flowing toward the tip grows because it is continually being reinforced by the field radiated from the base whose velocity in the direction of the wire is near the velocity of propagation of current along the wire (provided the bend angle is not too great). As a current pulse travels around a bend it is attenuated and distorted due to radiation [6]. Fig. 8 shows the signals obtained by subtracting various proportions of the tip reflection of a straight wire from the tip reflection of a bent wire. For $t < t_s$, the signals represent the current induced by radiation from the base. For $t > t_s$, the signals are a sum of the current induced by radiation and of the distorted current pulse traveling around the bend. The tail of the signal is smallest when 80 percent of the straight wire response is subtracted; this difference signal represents the induced current.

III. TRANSMISSION AND REFLECTION COEFFICIENTS FOR BENT WIRE ANTENNAS

The frequency response of an antenna can be inferred from a measurement of the pulse response [8], [11], [12], [16]–[19], or a part of the pulse response [20]. This procedure is called time-to-frequency transformation (TFT). The TFT technique can be used to obtain a transmission coefficient $\beta(f)$ that describes the coupling and attenuation of current as it propagates along the wire and a current reflection coefficient $\Gamma(f)$ that describes the reflection of current from the bend. In the simple network model of a bent wire antenna shown in Fig. 9, the base region, the bend, and the tip of the antenna are represented as lumped two port networks, which are described by their time domain scattering matrices $s_A(t)$, $s_B(t)$, and $s_C(t)$, respectively. The networks are joined by lossless and nondispersive transmission lines, which represent the straight sections of the wire.

Let $v_B(t)$ be the part of the pulse response that is the primary reflection from the bend, and $v_C(t)$ be the part that is the primary reflection from the tip. All parts have their time origins shifted to remove the delay of propagation along the wire. $v_B(t)$ and $v_C(t)$ are related to the test pulse by (where the asterisk denotes convolution)

$$v_B(t) = u(t) * s_{A_{12}}(t) * s_{B_{11}}(t) * s_{A_{21}}(t); \tag{1}$$

$$v_C(t) = u(t) * s_{A_{12}}(t) * s_{B_{12}}(t) * s_{C_{11}}(t) * s_{B_{21}}(t) * s_{A_{21}}(t). \tag{2}$$

Let $v_s(t)$ be the part of the pulse response that is the reflection

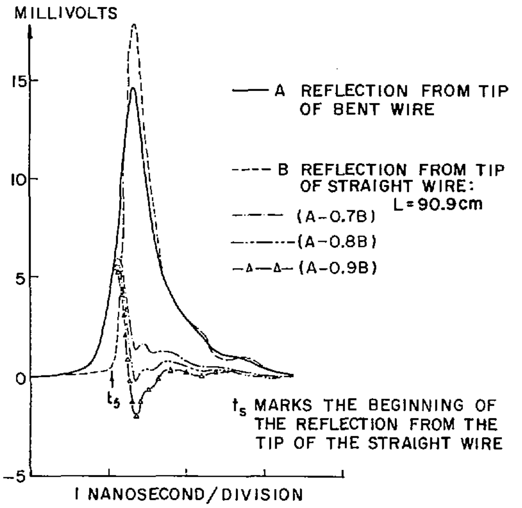

Fig. 8. Signal obtained by subtracting fraction of the attenuated tip reflection of straight wire from tip reflection of bent wire. For bent wire $D = 0.64$ cm, $r = 15$ cm, $\theta = 60°$, $L = 90.9$ cm, $l = 52.4$ cm.

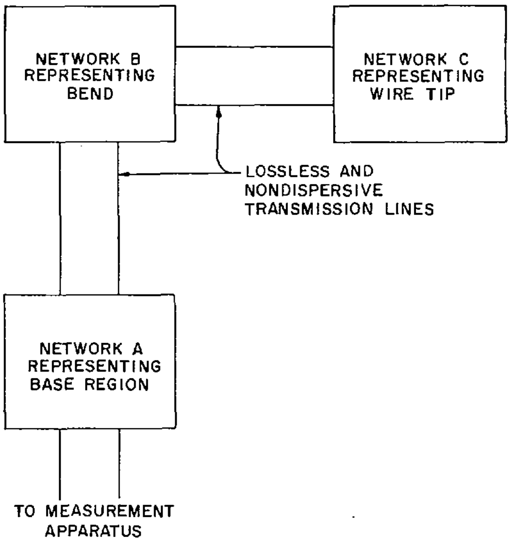
NETWORK B REPRESENTING BEND
NETWORK C REPRESENTING WIRE TIP
LOSSLESS AND NONDISPERSIVE TRANSMISSION LINES
NETWORK A REPRESENTING BASE REGION
TO MEASUREMENT APPARATUS

Fig. 9. Simple network model of bent wire antenna.

from the tip when the wire is straight, i.e., the network B is removed. The received part $v_s(t)$ is related to the test pulse by

$$v_s(t) = u(t) * s_{A_{12}}(t) * s_{C_{11}}(t) * s_{A_{21}}(t). \tag{3}$$

Knowing $s_{A_{12}}(t) = s_{A_{21}}(t)$, and taking the Fourier transforms of the scattering matrix coefficients in (1), (2), and (3), we obtain

$$[S_{B_{12}}(f)]^2 = \frac{V_C(f)}{V_S(f)}; \tag{4}$$

$$S_{B_{11}}(f) = \frac{V_B(f)}{V_S(f)} S_{C_{11}}(f). \tag{5}$$

The effective bandwidth of $v_s(t)$ is defined to be that band of frequencies of width W centered on F, within which $|V_S(f)|$ is well above the mean noise level. Thus

$$\beta(f) = S_{B_{12}}(f), \qquad F-(W/2)<|f|<F+(W/2); \tag{6}$$

and with the assumption that $S_{C_{11}}(f) = 1$ within the measurement bandwidth,

$$\Gamma(f) = -S_{B_{11}}(f), \qquad F-(W/2)<|f|<F+(W/2). \tag{7}$$

The minus sign denotes the change in direction of the reflected current. For the baseband pulse used in the measurements reported here, $F = 0$.

The reflection scattering coefficient of the tip $s_{C_{11}}(t)$ has been obtained by Ross [21] from a narrow pulse measurement. It is essentially real and equal to about 0.97 for the test pulse he used. Details were not given, but his experimental results suggest that the pulse duration was less than 500 ps, which means that its effective bandwidth extends to several GHz. Thus for computations of $\Gamma(f)$ up to several GHz, the assumption that $S_{C_{11}}(f) = 1$ should introduce less than 3 percent error.

The coefficients $\Gamma(f)$ and $\beta(f)$ can be used to relate the amplitude of a monochromatic current wave at the base of a bent wire antenna and traveling outwards on the antenna, to the amplitude of the monochromatic current wave arriving back at the base of the antenna. Thus, if $I_0(f)$ is the current at the base traveling outward along the wire and if $I_A(f)$ is the current at the base traveling inward, then

$$I_A(f) = I_0(f)\{\Gamma(f)e^{-j2kl} - [\beta(f)]^2 e^{-j2kL}\} \tag{8}$$

where l and L are defined in Fig. 1. The minus sign before the second term inside the braces accounts for the change in direction of the current when it is reflected from the wire tip.

When computing $\Gamma(f)$ and $\beta(f)$, $v_B(t)$, $v_C(t)$, and $v_S(t)$ were separated from the rest of the measured signal by applying an asymmetrical extended cosine bell data window [22], [8]. The phase of all the computed $\Gamma(f)$ is referred to the bend center. The phase error in $\Gamma(f)$ introduced by the means employed to do this [8] is estimated to be $< \pm 2.5°$ and $< \pm 12.5°$ at 2 GHz for the $r = 3$ mm and the $r = 0.64$ cm bends, respectively. Errors in the computed spectra due to aliasing and truncation [17] are better than 1 percent and 3 percent, respectively [8], error due to A/D quantizing, noise (using 20 μV as an estimate), and timing jitter (using 0.75 ps as an estimate) in $|\beta(f)|$ are insignificant, and better than 3.2 percent and 0.1 percent, respectively [23], [8]. Noise error is within ± 0.01 in $|\Gamma(f)|$ [8]. In Fig. 10, the erratic phase for small θ is attributed to experiment error.

The moduli of the spectra of the pulses reflected from the tips of the straight wires $|V_S(f)|$, which were used to compute the results presented in this section, are shown in Fig. 11. Computations of the frequency response deteriorate rapidly when $|V_S(f)|$ becomes smaller than -20 dB of its peak value [8]. This point defines the effective bandwidth of $v_s(t)$, and $W/2$ is thus about 3 GHz. The spectra of the pulses applied to the 0.32 cm diameter wires is different from the spectra of the pulses applied to the 0.64 cm diameter wires, in agreement with the earlier argument. Computed $\beta(f)$ and $\Gamma(f)$ for different wire geometries are plotted in Figs. 10–13.

IV. DISCUSSION

Fig. 10 shows that the modulus and the phase of the current reflection coefficients for sharp bends of a given bend angle θ increase with increasing frequency (the phase characteristic is nonlinear and does not correspond simply to a

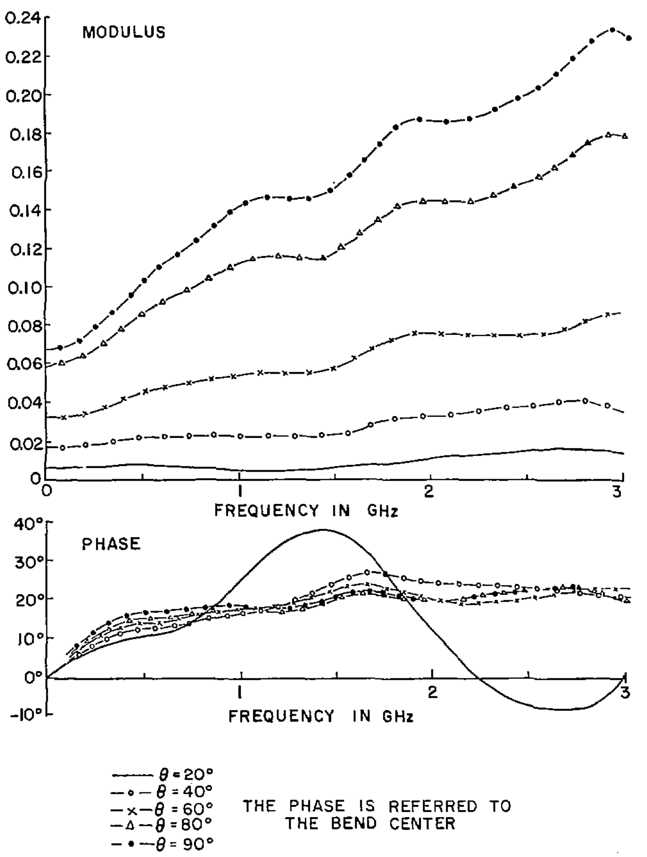

Fig. 10. Current reflection coefficients $\Gamma(f)$ for $r = 3$ mm bends in 0.32 cm diameter wires.

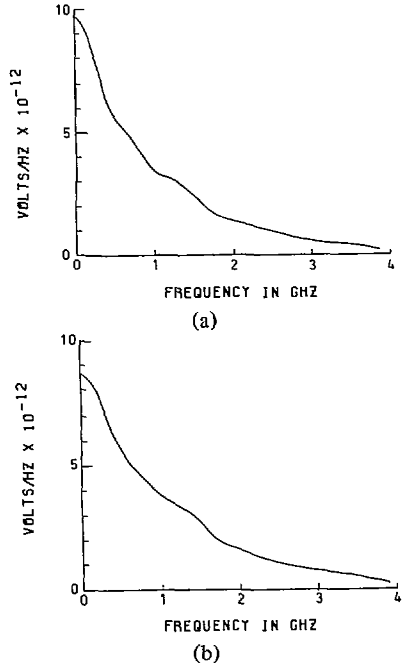

Fig. 11. (a) $|V_S(f)|$ used to compute transmission coefficients $\beta(f)$ shown in Fig. 12 and current reflection coefficients $\Gamma(f)$ shown in Fig. 13 (0.64 cm diameter wires); (b) $|V_S(f)|$ used to compute current reflection coefficients $\Gamma(f)$ shown in Fig. 10 (0.32 cm diameter wires).

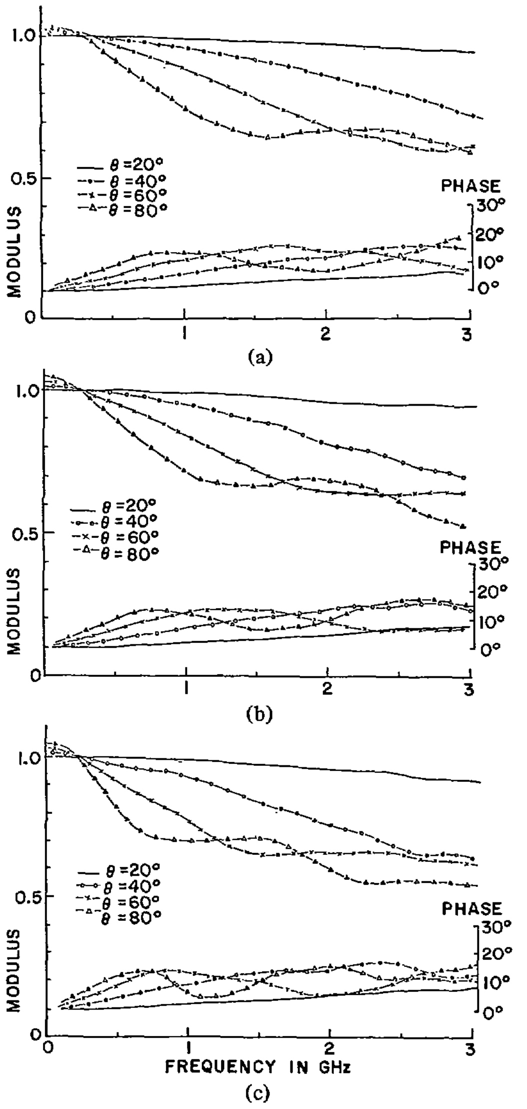

Fig. 12. Transmission coefficients $\beta(f)$ for $r = 15$ cm radius bends in 0.64 cm diameter wire antennas. (a) $L = 75.7$ cm, (b) $L = 90.9$ cm, and (c) $L = 121.3$ cm.

constant time delay). This causes compression of the reflected pulse. The phase characteristics indicate that the phase center of the reflection is near point P, but at 2 GHz, a phase advance of only about 6° would correspond to reflection from P. The phase error due to timing shift is less than 2.5° at 2 GHz, whereas the phase is observed to be about 22° at 2 GHz. This suggests that $S_{c_{11}}(f)$ is not real as assumed in the computations but has a phase angle of about +15°. $S_{c_{11}}(f)$ could be obtained simply by applying TFT procedures to Ross' experiment [21]. From Fig. 10, $|\Gamma(f)|$ at any frequency is approximately proportional to $\theta^{2.3}(0^\circ < \theta < 90^\circ)$. $\Gamma(f)$ for the gradual bends decreases with increasing frequency. The phase characteristics indicate that the phase center of the reflection is near P at low frequencies (i.e., when the bend is electrically sharp), and as the frequency increases the phase center of the reflection shifts towards the point Q.

The phase angles of all the computed $\beta(f)$ are positive because energy is coupled across the bend. Consider the wire for which $\beta(f)$ is shown in Fig. 12(c). The difference Δ between the direct path OZ (Fig. 1) and the path $OPQZ$ following the wire axis is 22.2 cm for a $\theta = 80^\circ$ bend corresponding to λ and

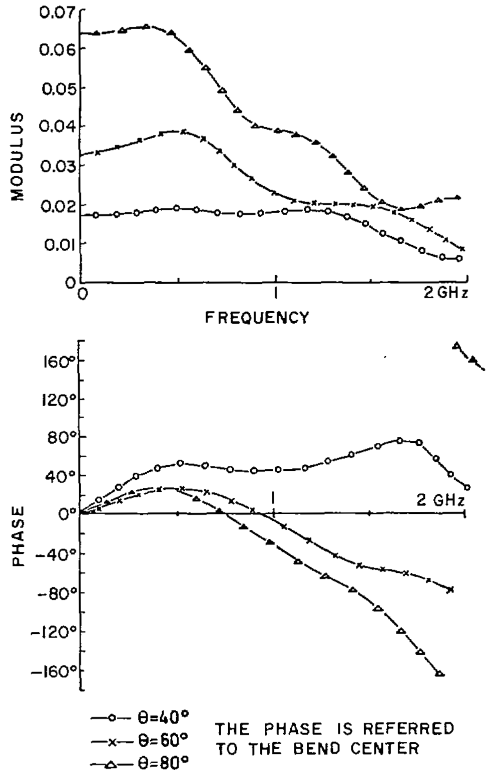

Fig. 13. Current reflection coefficients $\Gamma(f)$ for r = 15 cm bends in 0.64 cm diameter wires.

$\lambda/2$ at 1.35 GHz and 0.675 GHz, respectively. The induced current is in phase with the current traveling along the wire. The first minimum and maximum in $|\beta(f)|$ occur at frequencies slightly higher than those for which Δ is a half-wavelength and a wavelength, respectively, meaning that the phase center for reception corresponds to some point between P and Z, indicating that the base radiation is received by the whole length PQZ. All $\beta(f)$ are greater than unity at zero frequency because the average value of the signal reflected from the tip of the bent wire is higher than the average value of the signal reflected from the tip of a straight wire. Successive maxima and minima decrease in amplitude as the frequency increases, showing that attenuation, which increases with increasing frequency, occurs as current travels around the bend. Additional measurements [8] show that there is no significant change in $\beta(f)$ when the wire is fed from the opposite end and that there is significantly more attenuation of signals propagating around bends in thick wires than in thin wires. This latter observation is consistent with the known behavior of current on straight wires [7]. The computed $\beta(f)$ represent both the attenuation of the bend and reception of energy radiated from the feedpoint. Thus $\beta(f)$ does not simply describe the local effect of the bend, but it is a function of the whole wire and feedpoint geometry. The transfer function of the bend is implicit in the results.

Fig. 14 is a plot of a simple average of the $\beta(f)$ presented in Figs. 12(a) and (c). Because the bend geometry is identical for these two results but the total wire lengths are sufficiently different that for a particular frequency the base radiation subtracts from the bend response in one case and adds to it in the other, we can take the simple average presented in Fig. 14 as being an estimate of the true bend transfer function. Therefore Fig. 14 describes the relative amplitudes of the current waves

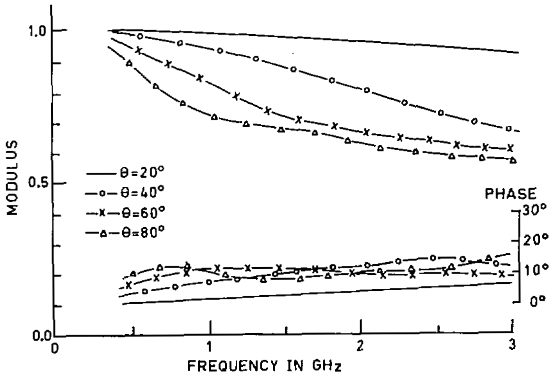

Fig. 14. Estimate of transmission coefficient $\beta(f)$ for r = 15 cm bends in 0.64 cm diameter wires. θ is bend angle.

on two infinitely long straight wires joined by the circular bend. The attenuation caused by the bend increases rapidly with increasing frequency until the arc length is about a half-wavelength. At higher frequencies, there is a less rapid increase in attenuation because the bend is less sharp electrically.

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A Hybrid Technique for Wire Antennas in a Cavity

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Abstract—Two numerical techniques for solving the current distribution on wire antennas, namely point matching and reaction matching, have been combined in a particular fashion to yield a new technique termed the "hybrid method." The hybrid method is well suited for a wire antenna mounted in a cavity. Over the antenna, piecewise sinusoidal basis functions with reaction matching are used, and on the support structure, point matching is used with pulse basis functions. The hybrid method has been applied to calculate the input impedance and the radiation efficiency of the three-turn loop antenna in an open circular cavity. Calculated input resistance of the antenna is found to agree quite well with measured data. Effects of varying the extent of recession of the antenna and varying the radius and the material of the antenna wire on the bandwidth, the efficiency, the antiresonance frequencies, and the general input reactance level of the antenna have been examined for a specific antenna geometry. The results are considered to be indicative of the general characteristics of a multiturn loop antenna with nearby support structure. The computer program that has been developed may be applied to a systematic parametric study of this type of antenna in order to optimize it for a particular application.

INTRODUCTION

TWO of the numerous numerical techniques and methods used for wire antennas and scattering bodies are reaction matching and point matching, both developed by Richmond at the Ohio State University ElectroScience Laboratory [1], [2]. These two methods have been extensively used in the solution of many practical antennas and scattering bodies [3]. Both of these methods require approximating a wire structure by a series of linear wire segments and a conducting surface by a linear wire grid. To arrive at the point matching solution, each wire segment is assumed to carry a uniform current and then the tangential component of the resultant electric field is forced to vanish at the midpoint of each segment. In reaction matching, however, a piecewise sinusoidal current distribution is assumed to exist over the wire segments, and then the reaction of a filamentary "test" dipole (put at the axis of each "true" dipole) with all the true dipoles of the antenna structure is equated to zero.

Recently, Harrington [4] unified the methods of many previous workers into a general concept which he termed "method of moments." Point matching is a special case of the moment method and uses pulse and delta functions as basis and testing functions, respectively. On the other hand, Richmond's reaction method [5] is a special case of Galerkin's method [3] using piecewise sinusoidal functions as basis and testing functions.

The current distribution yielded by the point matching method with pulse basis functions is a quantized distribution and does not satisfy Kirchhoff's current law at the junctions, while that given by the method of reaction matching is continuous with slope discontinuities at the junctions and satisfies the current law. Compared to the reaction technique, which uses a surface integral having contributions from the entire surface, the point matching technique enforces the boundary condition only at discrete points.

The fields of a wire segment carrying uniform current are not known in closed form, but the fields of a wire segment carrying sinusoidal current are. As a result, during the course of actual calculations, the method of point matching requires numerical integration and therefore more computer time, while the method of reaction does not require numerical integration (except for some special cases). In reaction matching, the large number of mode currents (number of mode currents at a junction is one less than the number of segments joining at that junction) makes it expensive to solve large structures on

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